

APPENDIX
TO LECTURE NOTES ON
ASSET PRICING AND PORTFOLIO
MANAGEMENT

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1. Portfolio analysis in Excel spreadsheet
2. Formula sheet
3. List of Additional Academic Articles

Notes on Portfolio Selection using Excel

These notes discuss how to find the optimal portfolio of risky assets using Excel (i.e. the point of tangency from the riskfree asset that gives an investor the best risk-return tradeoff).

The handout focuses on using vector notation to solve the problem. See BKM for a discussion of the problem.

I. Inputs

Main inputs: expected returns, variances and covariances. Standard (mathematical) way to summarize this information: vector of expected returns μ and variance-covariance matrix Σ .

The $(n \times 1)$ vector μ has elements which are the expected returns of each of the assets. It is usually better to work with the vector of excess expected returns μ^e , where the riskfree rate is subtracted from the expected returns.

The variance-covariance matrix Σ is an $(n \times n)$ matrix that records the variances in the diagonal (i.e. the elements Σ_{ii}), and the covariances in the off-diagonal terms (Σ_{ij}). This is a symmetric matrix ($\Sigma_{ij} = \Sigma_{ji}$), and it is negative-semidefinite (which means that when you run a portfolio x through it as $x^T \Sigma x$ you always get something that is non-negative (here superscript T denotes “transpose”).

Example 1. Expected return for three assets: 10%, 12%, 20%. Volatilities: 20% for all. Correlation between asset 1 and 2 is 0.50, and all other correlations are zero. Risk-free rate is 5%. Then

$$\mu^e = \begin{array}{l} 0.05 \\ 0.07 \\ 0.15 \end{array}$$

and

$$\Sigma = \begin{array}{ccc} 0.04 & 0.02 & 0 \\ 0.02 & 0.04 & 0 \\ 0 & 0 & 0.04 \end{array}$$

Note: $0.02 = 0.5(0.2)(0.2)$ is the covariance of assets 1 and 2.

II. Calculating means and variances

In order to calculate the expected excess return and variance of a portfolio x ,

$$\begin{aligned} E(r_x) - R_f &= x^T \mu_e \\ \text{var}(r_x) &= x^T \Sigma x \end{aligned}$$

Excel is a bit tricky in order to do vector and matrix operations. Best thing is to keep vectors vertical and matrices square. Then another useful trick is to hit <Ctrl><Shift><Enter> once a formula has been entered.

In order to perform the calculations, suppose the portfolio x is in cells H1:H3. If the variance-covariance matrix was in cells A1:C3, and the excess expected returns in E1:E3, the following two (intuitive) formulas would give as the expected excess return on x and its variance:

$$\begin{aligned} &=mmult(transpose(H1:H3),E1:E3) \\ &=mmult(mmult(transpose(H1:H3),A1:C3),H1:H3) \end{aligned}$$

Remember: you need to hit <Ctrl><Shift><Enter> once you entered the formula.

III. Optimal Portfolios

With no constraints on short-sales the solution to an investor's problem involves investing in the portfolio

$$x = \Sigma^{-1} \mu^e \quad (1)$$

End of story.

Example 2. Equation (1) tells us that the optimal portfolio (an $n \times 1$ vector) is equal to the product of the inverse of Σ and μ^e .

In Excel, when performing an operation involving vectors and/or matrices you need to select the set of cells that are the solution before entering the formula (you did not need this before because the mean and variance of a portfolio were scalars, i.e. 1×1 vectors).

In our particular case this involves selecting a 3×1 area of cells. If the variance-covariance matrix was in cells A1:C3, and the excess expected returns in E1:E3, the command that gives you the answer is (don't press Enter yet) is

$$=mmult(minverse(A1:C3), E1:E3)$$

after which we need to press: <Ctrl><Shift><Enter> (don't ask me why).

It's tricky following these steps the first time, but it is worth the time.

It's also worth remembering that this is an intermediate step in the asset allocation process. Once we know what the optimal risky portfolio x is, we still need to decide on the mix between the riskfree asset and this optimal risky portfolio.

IV Alternative Approach

Section 8.5 in BKM discusses another approach to constructing the mean-variance efficient frontier.

Note that in the previous discussion, we did not look for the whole frontier, but rather just for the optimal portfolio among those on the frontier (which will be combined with the riskfree asset).

It is not surprising that a similar spreadsheet as that in 8.5 should help us get the job done using Solver. As we should all know, the optimal risky portfolio solves

$$\max(x) S_x = [E(r_x) - R_f]/\sigma_x = (x^T \mu^e) / \sqrt{x^T \Sigma x} \quad (2)$$

We know how to calculate the quantities in the numerator and denominator, so we can just use Solver.

It should be noted that using this “brute-force” approach by maximizing (2) is the only way to find a solution once we include constraints (such as short-sale constraints).

Formula Sheet

- **Conversion between continuously compounded (cc) and simple (s) returns:**

$$r_s = \exp(r_{cc}) - 1 \quad r_{cc} = \ln(1+r_s)$$

- **Fraction of your wealth you put in the risk-free asset A :**

$$w^* = E(r_A - r_F) / A \sigma_A^2$$

- **The MVE portfolio weights when there are two risky assets ($x_2 = 1 - x_1$)**

$$x_A = [E(r_A^e) \sigma_B^2 - E(r_B^e) \sigma_{AB}^e] / \{ [E(r_A^e) \sigma_B^2 + E(r_B^e) \sigma_A^2] - [E(r_A^e) + E(r_B^e)] \sigma_{AB}^e \}$$

- **The CAPM equation**

$$E(r_i) = r_F + \beta_i [E(r_M) - r_F] \quad \text{where } \beta_i = \sigma_{im} / \sigma_M^2$$

- **The security characteristic line**

$$r_{it} - r_{Ft} = \alpha_i + \beta_i (r_{mt} - r_{Ft}) + \varepsilon_{it}$$

- **The systematic variance of a security**

$$\text{CAPM: } \sigma_{sys,i}^2 = \beta_i^2 \sigma_M^2 \quad (\text{CAPM})$$

$$\text{APT: } \sigma_{sys,i}^2 = \sum_j \sum_k \beta_{ij} \beta_{ik} \sigma_{jk} = \sum_k \beta_{ik}^2 \sigma_k^2 \quad \text{when factors are uncorrelated}$$

- **The variance of portfolio a and covariance of portfolios a and b**

$$\sigma_a^2 = \sum_i \sum_j x_i^a x_j^a \sigma_{ij} \quad \sigma_{ab} = \sum_i \sum_j x_i^a x_j^b \sigma_{ij}$$

Two assets, 1 and 2:

$$\sigma_a^2 = (x_1^a)^2 \sigma_1^2 + (x_2^a)^2 \sigma_2^2 + 2x_1^a x_2^a \sigma_{12}$$

$$\sigma_{ab} = x_1^a x_1^b \sigma_1^2 + x_2^a x_2^b \sigma_2^2 + (x_1^a x_2^b + x_2^a x_1^b) \sigma_{12}$$

- **The APT return generating process**

$$r_{it} = E(r_{it}) + \beta_{i1}f_{1t} + \beta_{i2}f_{2t} + \dots + \beta_{ik}f_{kt} + \varepsilon_{it}$$

- **The APT pricing equation**

$$E(r_i) = \lambda_0 + \lambda_1\beta_{i1} + \lambda_2\beta_{i2} + \dots + \lambda_k\beta_{ik}$$

- **Managed fund performance measures**

(1) The Sharpe Ratio of portfolio p

$$S_p = r_p^e / \sigma_p$$

(2) Jensen's Alpha is the α_p from the regression

(CAPM) $r_{pt}^e = \alpha_p + \beta_p r_{mt}^e + \varepsilon_{pt}$

(APT) $r_{pt}^e = \alpha_p + \beta_{p1} r_{1t}^e + \beta_{p2} r_{2t}^e + \dots + \beta_{pK} r_{Kt}^e + \varepsilon_{pt}$

where r_{kt}^e is the excess return on the factor-mimicking portfolio for factor k

(3) The Treynor Measure

$$T_p = (E(r_p) - r_F) / \beta_p$$

or, adjusted (relative to the slope of the SML)

$$T_p^* = (E(r_p) - r_F) / \beta_p - (E(r_M) - r_F) / \beta_M = \alpha_p / \beta_p$$

(4) The Appraisal Ratio of portfolio p (CAPM and APT)

$$AR_p = \alpha_p / \sigma_{ep}$$

Sharpe Ratio of optimal portfolio C of M and p is

$$SR_C = (SR_M^2 + AR_p^2)^{1/2}$$

(5) The Henriksen-Merton Timing Measure is c_p from the regression

$$r_{pt}^e = \alpha_p + \beta_p r_{Mt}^e + D_t c_p r_{Mt}^e + \varepsilon_{pt}$$

where D_t is a dummy variable that takes on a value of 1 when $r_{Mt} > r_{Ft}$

(6) The Treynor-Mazuy Timing Measure is the coefficient c_p from the regression

$$\text{(CAPM)} \quad r_{pt}^e = \alpha_p + \beta_p r_{Mt}^e + c_p (r_{Mt}^e)^2 + \varepsilon_{pt}$$

$$\text{(APT)} \quad r_{pt}^e = \alpha_p + \beta_{p1} r_{1t}^e + \beta_{p2} r_{2t}^e + \dots + \beta_{pK} r_{Kt}^e + c_{p1} (r_{1t}^e)^2 + c_{p2} (r_{2t}^e)^2 + \dots + c_{pK} (r_{Kt}^e)^2 + \varepsilon_{pt}$$

The value of *selectivity* is α_p

CAPM: The value of *market timing* is $c_p \sigma_M^2$

APT: The total value of *factor tilting* is $c_{p1} \sigma_1^2 + c_{p2} \sigma_2^2 + \dots + c_{pK} \sigma_K^2$

List of Academic Articles

I. Stock Market Survival

1. Jorion and Goetzmann: “Global Stock Markets in the Twentieth Century”, *Journal of Finance*, 1999

2. Aggarwal and Angel: “The Rise and Fall of the Amex Emerging Company Marketplace”, *Journal of Financial Economics*, 1999

II. Asset Pricing Basics

3. Campbell: “Asset Pricing at the Millennium”, *Journal of Finance*, 2000

4. Fama and French: “Multifactor Explanations of Asset Pricing Anomalies”, *Journal of Finance* 1996

5. Lamont: “Earnings and Expected Return”, *Journal of Finance*, 1998

III. Return to Momentum Strategies

6. Rouwenhorst: “International Momentum Strategies”, *Journal of Finance*, 1998

7. Moskowitz and Grinblatt: “Do Industries Explain Momentum?”, *Journal of Finance*, 1999

IV. Management Fees in Closed-End Funds

8. Coles, Suay and Woodbury: “Fund Advisor Compensation in Closed-End Funds”, *Journal of Finance*, 2000

V. Mutual Fund Performance

9. Wermers: “Mutual Fund Performance: An Empirical Decomposition into Stock-Picking Talent, Style, Transactions Costs, and Expenses”, *Journal of Finance* 2000

10. Zheng: “Is Money Smart? A Study of Mutual Fund Investors’ Fund Selection Ability”, *Journal of Finance*, 1999

11. Keim: “An Analysis of Mutual Fund Design: The Case of Investing in Small-Cap Stocks”, *Journal of Financial Economics*, 1999

VI. Performance and Flow of Funds

12. Sirri and Tufano: “Costly Search and Mutual Fund Flows”, *Journal of Finance*, 1998

13. Jain and Wu: “Truth in Mutual Fund Advertising: Evidence on Future Performance and Fund Flows”, *Journal of Finance*, 2000

VII. Use of Derivatives by Mutual Funds

14. Koski and Pontiff: “How are Derivatives Used? Evidence from the Mutual Fund Industry”, *Journal of Finance*, 1999

VIII. Closed-End Fund Discounts

15. Pontiff: “Closed-End Fund Premia and Returns: Implications for Financial Market Equilibrium”, *Journal of Financial Economics*, 1995

IX. Hedge Funds

16. Fung and Hsieh: “A Primer on Hedge Funds”, *Journal of Empirical Finance*, 1999

17. Fung and Hsieh: “Measuring the Market Impact of Hedge Funds”, *Journal of Empirical Finance*

18. Ackermann, McNally and Ravenscraft: “The Performance of Hedge Funds: Risk, Return and Incentives”, *Journal of Finance*, 1999

X. Private Equity

19. Gompers and Lerner: “Money Chasing Deals? The Impact of Fund Inflows on Private Equity Valuations”, *Journal of Financial Economics*, 2000

XI. Pension Fund Activism

20. Guercio and Hawkins: “The Motivation and Impact of Pension Fund Activism”, *Journal of Financial Economics*, 1999

XII. Individual Investor Portfolios

21. Barber and Odean: “Trading is Hazardous to your Wealth: The Common Stock Investment Performance of Individual Investors”, *Journal of Finance*, 2000

XIII. Performance of New Issues

22. Eckbo, Masulis and Norli “Seasoned Public Offerings: Resolution of the ‘New Issues Puzzle’”, *Journal of Financial Economics*, 2000

23. Eckbo and Norli: “Leverage Liquidity and Long-Run IPO Returns”, working paper Tuck School at Dartmouth.

XIV. Performance of Insider Trades

24. Eckbo and Smith: “The Conditional Performance of Insider Trades”, *Journal of Finance*, 1998